Roll No. ..............................

# 21253

## B. Sc. (Hons.) Maths 2nd Semester Examination – May, 2019

### VECTOR CALCULUS

Paper : BHM-123

*Time : Three hours ]* *[ Maximum Marks : 60*

*Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.*

**Note :** Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. **9** (Section – V) is ***compulsory***. All questions carry equal marks.



### SECTION – I

1. (a) Show that the vectors :

$\vec{a}-2\vec{b}+3\vec{c}, -2\vec{a}+3\vec{b}-4\vec{c}$ and $\vec{a}-3\vec{b}+5\vec{c}$ are coplaner.

(b) Show that :

$$(\vec{b}\times\vec{c})\times(\vec{a}\times\vec{d})+(\vec{c}\times\vec{a})\times(\vec{b}\times\vec{d})+(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})$$
$$=-2\left[\vec{a}\ \vec{b}\ \vec{c}\right]\ \vec{d}$$

P. T. O.



2. (a) The necessary and sufficient condition for the vector function $\vec{f}$ of a scalar variable $t$ to have a constant magnitude is $\vec{f}\cdot\frac{d\vec{f}}{dt}=0$.

(b) A particle moves along the curve $x=t^3+1, y=t^2, z=2t+5$, where $t$ is time. Find the components of velocity and acceleration at $t=1$ in the direction of $\hat{i}+\hat{j}+3\hat{k}$.

### SECTION – II

3. (a) For any vector $\vec{a}$, show that $\nabla(\vec{a}\cdot\vec{r})=\vec{a}$, where $\vec{r}$ is the position vector of a point. Hence show that grad $\left[\vec{r}\ \vec{a}\ \vec{b}\right]=\vec{a}\times\vec{b}$.

(b) Given the curve of intersection of two surfaces $x^2+y^2+z^2=1$ and $x+y+z=1$; Find the tangent line at the point (1, 0, 0).

4. (a) If div $(\phi(r)\vec{r})=0$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and $\left|\vec{r}\right|=r$, then prove that $\phi(r)=\frac{C}{r^3}$.

(b) Prove that :

$$\nabla\times(\vec{f}\times\vec{g})=\vec{f}(\nabla\cdot\vec{g})-\vec{f}\cdot(\nabla\vec{g})+\vec{g}\cdot(\nabla\vec{f})-\vec{g}(\nabla\cdot\vec{f})$$

### SECTION – III

5. (a) If $u, v, w$ are orthogonal curvilinear co-ordinates, then $\frac{\partial\vec{r}}{\partial u},\frac{\partial\vec{r}}{\partial v},\frac{\partial\vec{r}}{\partial w}$ and $\nabla u, \nabla v, \nabla w$ are reciprocal system of vectors.

## SECTION – V

Show that the four points with position vectors $4\hat{i}+5\hat{j}+\hat{k}, 3\hat{i}+9\hat{j}+4\hat{k}, -\hat{j}-\hat{k}$ and $4(-\hat{i}+\hat{j}+\hat{k})$ are coplaner.

Prove that :

$$\frac{\vec{a}\times d^2b^{-1}}{dt^2}-\frac{d^2a^{-1}}{dt^2}\times\vec{b}=\frac{d}{dt}\left(\vec{a}\times\frac{d\vec{b}}{dt}-\frac{d\vec{a}}{dt}-\frac{d\vec{a}}{dt}\times\vec{b}\right)$$

If :

$\vec{a}=\sin\theta\,\hat{i}$

$\vec{b}=\cos\theta\,\hat{i}-$

$\vec{c}=2\hat{i}+3\hat{j}$ -

find $\frac{d}{d\theta}[\vec{a}\times$ ... at $\theta=\frac{\pi}{2}$.

) Find $\lambda,\mu,\upsilon$ ... ector :

$(2x+3y+\lambda z)$ ... $+3z)\ \hat{j}+(2x+\upsilon y+3z)\hat{k}$

is irrotation

) Find curl (f ... e function :

$$\vec{f}=y(x+z)\hat{i}+z(x+y)+x(y+z)\hat{k}.$$

) Define volume integral.

---